

9/11/17 "Success is a journey, not a destination."-Ben Sweetland

HW: ~~Test~~ #1-4 on homework section  
Test 1 on Tuesday 9/19

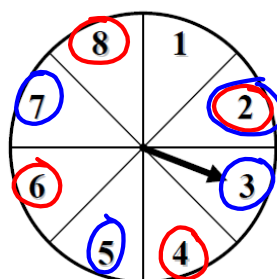
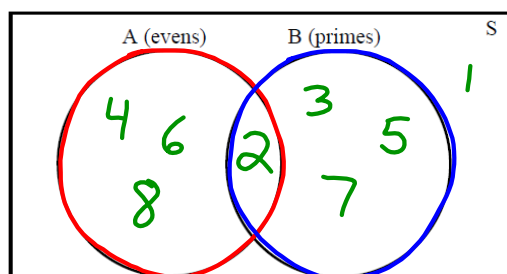
AIM: How do we add probabilities?

Warm Up:

There are times that we want to determine the probability that either event A happened or event B happened. To do this, we need to be able to account for all of the outcomes that fall into either one of the two events. Let's see how this looks given a simple Venn diagram.

**Exercise #1:** Consider the spinner shown below that has been divided into eight equally sized sectors of a circle. The spinner is spun once. In this experiment we will let A be the event of it landing on an even and B be the event of it landing on a prime number.

Fill in the Venn Diagram below with the actual numbers from the spinner.



When we have two (or more) sets, we can talk about their **union** and their **intersection**. Their technical definitions are given below.

THE UNION AND INTERSECTION OF TWO SETS	
For two sets, A and B, their <b>union</b> , OR, and their <b>intersection</b> , AND, are given by:	
(1) <b>Union:</b>	(2) <b>Intersection:</b>
A or B = $A \cup B = \{x : x \text{ is in A or } x \text{ is in B}\}$	A and B = $A \cap B = \{x : x \text{ is in A and } x \text{ is in B}\}$

**Exercise #2:** From Exercise #1 write out the following two sets:

(a) A or B (The Union):

$$A \text{ or } B = \{2, 3, 4, 5, 6, 7, 8\}$$

(b) A and B (The Intersection):

$$A \text{ and } B = \{2\}$$

$$7 = 4 + 4$$

**Exercise #3:** From Exercise #2, why is the equation  $n(A \text{ or } B) = n(A) + n(B)$  generally *not* true? What would be the correct modification to make it true? Use the last example to help explain.

$$n(A) = 4$$

$$n(B) = 4$$

$$n(A \text{ or } B) = 7$$

$$n(A \text{ or } B) = n(A) + n(B) - n(A \cap B)$$

$$= 4 + 4 - 1$$

$$n(A \text{ or } B) = 7$$

Two-way frequency charts give us a great example of how **events or sets can combine (union) and overlap (intersection)**. Let's take a look at this and develop some ideas about probability along the way.

**Exercise #4:** A small high school surveyed 52 of its seniors about their plans after they graduate. They found the following data and wanted to analyze it based on gender. In this case, if we pick a student at random we can place them into one of four events:

M = Male

F = Female

C = Going to College

N = Not going to college

	Gender		Total
	Male	Female	
Going to College	16	13	29
Not Going to College	14	9	23
Total	30	22	52

(a) Give the values for each of the following:

(i)  $n(M) = 30$

(ii)  $n(F) = 22$

(iii)  $n(C) = 29$

(iv)  $n(N) = 23$

(v)  $n(M \text{ and } C) = 16$

(vi)  $n(F \text{ and } C) = 13$

(vii)  $n(F \text{ or } C) = 22 + 29 - 13 = 38$

(b) What is the probability that a person picked at random would be a female who is going to college? Represent this using either a union or an intersection.

$$\frac{\text{Want}}{\text{Total}} = \frac{13}{52}$$

(c) What is the probability that a person picked at random would be a female or someone going to college? Represent this using either a union or an intersection.

$$P(F \text{ or } C) = \frac{38}{52}$$

(d) Explain why  $P(F \text{ or } C) \neq P(F) + P(C)$ ?

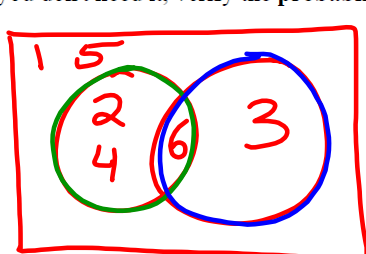
There are females that go to college (overlap)

(e) Fill in the general probability law based on (d):

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Sometimes we can avoid the probability law that we encounter in (e) by simply keeping careful track of what elements of the sample space are in both of our sets and making sure we don't count any element twice.

**Exercise #5:** A standard six-sided die is rolled once. Find the probability that the number rolled was either an even or a multiple of three. Represent this problem and the sets involved using a Venn diagram. Even though you don't need it, verify the **probability addition rule** from Exercise #4 (e).



$$P = \frac{4}{6}$$

$$P(\text{OR}) = P(E) + P(M \text{ of } 3) - P(\text{Both}) = \frac{3}{6} + \frac{2}{6} - \frac{1}{6} = \frac{4}{6}$$

There are some situations, though, where the **probability addition rule** is unavoidable.

**Exercise #6:** Insurance companies typically try to sell many different policies to the same customers. At one such company, 56% of all of the customers have car insurance policies, 48% have home insurance policies, and 18% have both. A customer is picked at random.

(a) Find the probability that she or he has at least one of the policies.

(b) Find the probability that she or he has neither of the policies.

1. Given the two sets below, give the sets that represent their union and their intersection.

$$A = \{3, 5, 7, 9, 11, 13\}$$

$$B = \{1, 5, 9, 13, 17\}$$

(a) Union: A or B =

(b) Intersection: A and B =

$$\{1, 3, 5, 7, 9, 11, 13, 17\}$$

8 elements

$$\{5, 9, 13\}$$

2. Using sets A and B from #1, verify the addition law for the union of two sets:

$$n(A \text{ or } B) = n(A) + n(B) - n(A \text{ and } B)$$

$$= 6 + 5 - 3$$

$$n(A \text{ or } B) = 8$$

- Red Hook High School has 480 freshmen. Of those freshmen, 333 take Algebra, 306 take Biology, and 188 take both Algebra and Biology. Which of the following represents the number of freshmen who take at least one of these two classes?

(1) 639

(3) 451

(2) 384

(4) 425

What is the number taking Bio or Algebra?

$$n(\text{Alg or Bio}) = n(\text{Alg}) + n(\text{Bio}) - n(\text{Both})$$

$$= 333 + 306 - 188$$

$$n(\text{Alg or Bio}) = 451$$

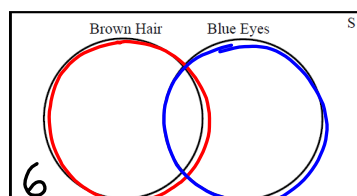
4. Evie was doing a science fair project by surveying her biology class. She found that of the 30 students in the class, 15 had brown hair and 17 had blue eyes and 6 had neither brown hair nor blue eyes. Determine the number of students who had brown hair and blue eyes. Use the Venn Diagram below to help sort the students if needed.

$$30 - 6 = 24$$

incircles

$$\text{Union} = 24$$

OR



$$n(\text{Brown or blue}) = n(\text{Brown}) + n(\text{Blue}) - n(\text{Both})$$

$$24 = 15 + 17 - x$$

$$24 = 32 - x$$

$$\begin{array}{r} -32 \\ -32 \\ \hline \end{array}$$

$$-8 = -x$$

$$8 = x$$

8 students  
Brown hair  
AND  
blue eyes

5. A standard six-sided die is rolled and its outcome noted. Which of the following is the probability that the outcome was less than three or even?

(1)  $\frac{2}{3}$

(3)  $\frac{5}{6}$

(2)  $\frac{1}{3}$

(4)  $\frac{1}{6}$

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6. Historically, a given day at the beginning of March in upstate New York has a 18% chance of snow and a 12% chance of rain. If there is a 4% chance it will rain and snow on a day, then which of the following represents the probability that a day in early March would have either rain or snow?

(1) 0.30

(3) 0.02

(2) 0.34

(4) 0.26

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7. A survey was done of students in a high school to see if there was a connection between a student's hair color and her or his eye color. If a student is chosen at random, find the probability of each of the following events.

- (a) The student had black hair.

- (b) The student had blue eyes.

- (c) The student had brown eyes and black hair.

		Hair Color			Total
		Black	Blond	Red	
Eye Color	Blue	0.15	0.20	0.05	0.40
	Brown	0.25	0.10	0.00	0.35
	Green	0.05	0.05	0.15	0.25
	Total	0.45	0.35	0.20	1.00

- (d) The student had blue eyes or blond hair.

- (e) The student had black hair or blue eyes.